SLOW-WAVE PROPERTIES OF SUPERCONDUCTING MICROSTRIP TRANSMISSION LINES

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ABSTRACT

A complex resistive boundary condition is used to accurately model very thin superconducting films used in microstrip transmission lines. The imaginary part of the conductivity is a measure of the energy stored in the superconductor which contributes to the slow-wave propagation behavior of these transmission lines. Numerical solutions of superconducting microstrip have been obtained and the dependence of the complex propagation constant on the microstrip geometry and the superconducting thin film properties was investigated.

INTRODUCTION

Transmission lines using superconducting films have many possible practical applications in microwave and millimeter-wave devices and circuits [1]. Advantages of superconducting transmission lines include low loss and low dispersion. In addition, in a superconducting microstrip configuration where the superconducting film thicknesses are small compared to the superconducting penetration depth (λ) the phase velocity can be very dependent on the superconducting properties of the strip and ground plane. The penetration depth is the characteristic decay length of the magnetic field into a superconductor. A reduction in the phase velocity is the result of the kinetic inductance (L_k) contribution from the energy stored as kinetic motion of the charge carriers in the superconductor.

Microstrip transmission lines, with a geometry designed to insure that L_k is much larger than the magnetic inductance (L_m), have been fabricated and found to have very slow phase velocities [2,3]. Due to the extreme geometry involved in such structures (the dielectric is a sputtered thin film less than 1000 Å), the behavior can accurately be predicted by a parallel plate waveguide model. However, the parallel plate approximation is inaccurate for more conventional microstrip geometries associated with substrate thicknesses on the order a mil thick and with microstrip widths of approximately the same dimension.

Traditional numerical approaches are not applicable to this situation since the superconducting films are much thinner than λ. For superconducting film thicknesses (t) much less than λ the fields can penetrate through the superconductor [4,5]. To address this case, very thin superconducting films ($t \ll \lambda$) can be modeled as a complex resistive boundary condition [6]. This paper is an extension of that work, which presented the analytical and numerical development and compared these results to earlier experimental work [2,3]. This paper summarizes the numerical approach and presents some of the results of a numerical investigation of the dependence of the propagation coefficient on several material and geometric parameters.

NUMERICAL APPROACH

The resistive boundary condition is an approximate boundary condition in electromagnetic theory [7]. It has been used to treat thin lossy dielectric sheets with large conductivities where the thickness is much less than a wavelength. The resistance, in Ω/square, of the analogous superconducting sheet of thickness t_{sc} is given by

$$R = [t_{sc}(\sigma_n - j\sigma_{sc})]^{-1} \quad . \qquad (1)$$

If the two-fluid model [8] is used, then σ_n, which is related to the losses in the superconducting film, is the conductivity of the normal electrons and is given by

$$\sigma_n = \sigma_{nc}(T/T_c)^4 \quad , \qquad (2)$$

where σ_{nc} is the normal state conductivity just above the transition temperature (T_c). The imaginary part of the conductivity accounts for the energy stored in the film and is given by

$$\sigma_{sc} = [1-(T/T_c)^4]/(\omega\mu_0\lambda_0{}^2) \; , \qquad (3)$$

where λ_0 is the penetration depth at 0.0 K. The strong temperature dependence of σ_n and σ_{sc} as $T \to T_c$ should be noted.

Solving the electromagnetic field problem for such a sheet can be reduced from a problem of matching fields at two surfaces to matching fields at one surface, under the restriction that $\lambda >> t_{sc}$, if it is assumed that

$$\lim_{\substack{t_{sc}\to 0 \\ \sigma\to\infty}} [\sigma t_{sc}]^{-1} = R \qquad (4)$$

The numerical formulation is an extension of the approach used for perfectly conducting microstrip using a full-wave approach [9]. The current components on the strip (J_x and J_y) for the geometry shown in Fig. 1 give rise to a set of coupled integral equations, which in the Fourier transform domain are

$$\tilde{Z}_{yy}(\zeta)\tilde{J}_y(\zeta) + \tilde{Z}_{yz}(\zeta)\tilde{J}_z(\zeta) = R\tilde{J}_y(\zeta) + \tilde{E}_y^e(\zeta) \qquad (5)$$

and

$$\tilde{Z}_{zy}(\zeta)\tilde{J}_y(\zeta) + \tilde{Z}_{zz}(\zeta)\tilde{J}_z(\zeta) = R\tilde{J}_z(\zeta) + \tilde{E}_z^e(\zeta) \qquad (6)$$

where $\tilde{Z}_{yy}$, $\tilde{Z}_{yz}$, $\tilde{Z}_{zy}$ and $\tilde{Z}_{zz}$ are transformed impedance Green's function elements which are determined using the spectral domain immitance approach [10]. $E_z^e(y)$ and $E_y^e(y)$ are the electric field components in the plane of but external to the strip. The $\tilde{}$ above the quantity denotes the Fourier transform of that quantity and ζ is the transform variable of the y dimension of Fig. 1. In the perfectly conducting case (R = 0) (5) and (6) are exactly equivalent to previously published approaches for a strip with a perfectly conducting boundary condition [9,10]. Thus, by modifying two of the transformed impedance Green's function elements by subtracting R, problems of interest can be solved using established numerical techniques. The currents are expanded in a set of basis functions. A Galerkin approach [11] is used to construct a determinantal equation for the

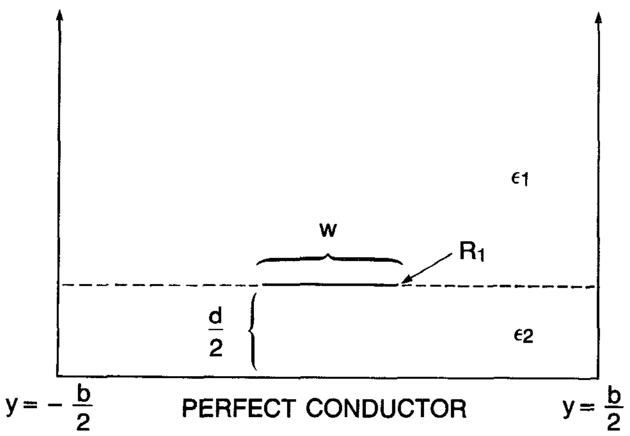

Figure 1. Geometry of the problem which was numerically implemented. A strip with a resistive boundary condition, R_1, is separated from a perfectly conducting ground plane by a dielectric, ε_2, of thickness d/2.

unknown propagation constant, a. The computer code is a modified version of the computer code from Ref. 12.

RESULTS

The application of the resistive boundary condition to model transmission lines consisting of thin superconducting films using this numerical technique was shown [6] to accurately determine the propagation constant of these structures. Fig. 2 summarizes those results by comparing the experimental, analytical (parallel plate), and numerical values which can be seen to be in very good agreement. The values presented are for the normalized propagation coefficient of the dominant mode, a', which is given by

$$a' = a/[\omega(\mu_0\varepsilon_0)^{1/2}] \quad . \qquad (7)$$

Thus the Re(a') represents the slowing factor compared to an equivalent perfectly conducting suspended stripline with a substrate having a dielectric constant of ε_0. The experimental results were obtained from microstrip line which was fabricated from a 150 Å niobium nitride ground plane, a 400 Å silicon dielectric, and a 140 Å thick niobium nitride strip which is 25 μm wide [2,3]. The parameters of the superconducting films of this line were found to be [3]; λ_0 = 320 nm and T_c = 12.15 K. The analytical and numerical results presented in Fig. 2 were obtained by modeling the experimental geometry.

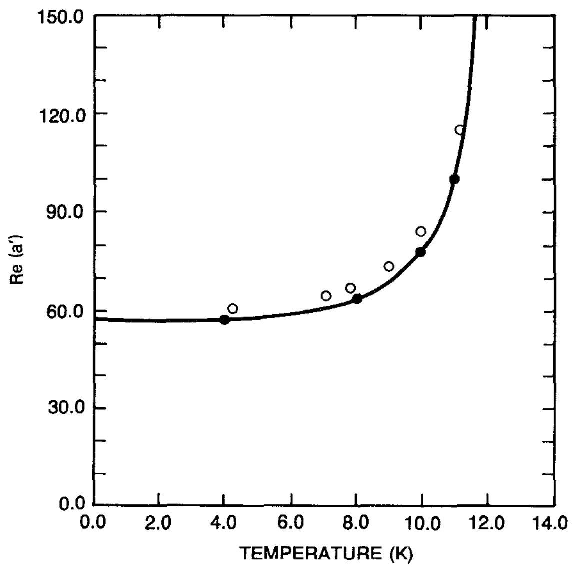

Figure 2. A comparison of the experimental (o), numerical (•) and analytical (——) results as a function of temperature is shown for geometries which most closely approximate the experimental case.

Even when the dielectric is on the order of a mil thick the propagation constant can still be dependent on the energy stored in the superconducting film. The dependence of a' on frequency is shown in Fig. 3 for two 25 μm wide strips separated by a 25 μm thick substrate with a relative dielectric constant, ε_r, of 10.5. The superconducting film parameters assumed were T_c = 12.15 K, T = 4.0 K, t_{sc} = 145 Å, λ_0 = 320 nm, and $\sigma_{nc} = 0.5\times10^6\ (\Omega\text{-cm})^{-1}$. As expected from the geometry, since all dimensions are small compared to a wavelength, Re(a') is, for all practical purposes, independent of frequency. However, the superconductor losses, and hence Im(a') are linearly dependent on frequency in this frequency band.

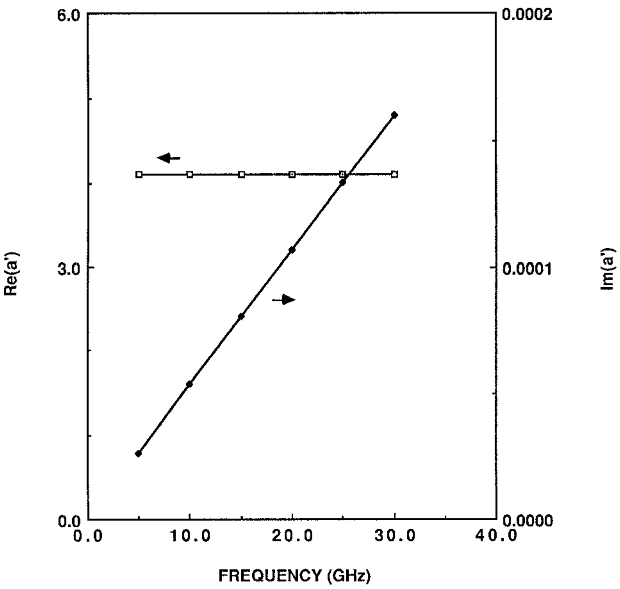

Figure 3. The dependence of Re(a') and Im(a') on frequency is shown at T = 4.0 K for 25 μm wide strips separated by a 25 μm substrate with a relative dielectric constant, ε_r = 10.5. The superconducting parameters are T_c = 12.15 K, $\sigma_{nc} = 0.5\times10^6\ (\Omega\text{-cm})^{-1}$, t_{sc} = 145 Å and λ_0 = 320 nm.

The strong functional dependence of the complex conductivity on temperature as $T \rightarrow T_c$ given by (2) and (3) yields a very strong functional dependence of the complex propagation constant on temperature. For the geometry and material parameters described in the above paragraph, Fig. 4 shows the dependence of Re(a') and Im(a') at 10.0 GHz as a function of temperature. While the slowing factor increases by about 50 % over the temperature range from 2.0 K to 11.0 K, Im(a'), which represents the loss due to the normal electrons, increases by over three orders of magnitude. The value of Re(a') for the perfectly conducting case is about 2.73 for the same geometry. The difference between the Re(a') in these two cases is due to the kinetic energy stored in the superconducting film which for this geometry is on the same order of magnitude as the magnetic energy stored in the dielectric. The dependence of Re(a'), at 10 GHz for a 25 μm wide suspended stripline, on the dielectric substrate thickness is shown in Fig. 5

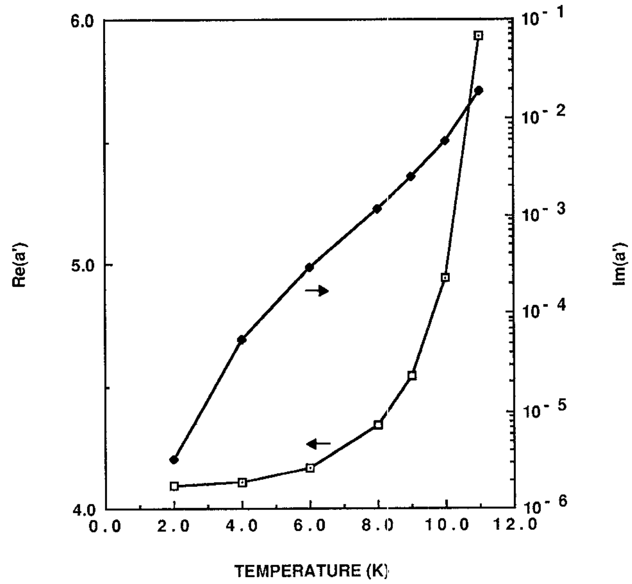

Figure 4. The dependence of Re(a') and Im(a') at 10.0 GHz on temperature is shown for two 25 μm wide strips separated by a 25 μm thick substrate with a relative dielectric constant, ε_r, of 10.5. The superconducting parameters are the same as Fig. 3.

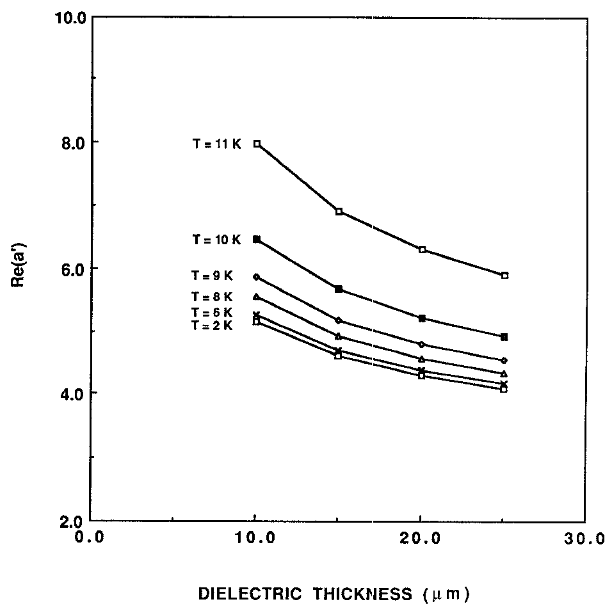

Figure 5. The dependence of Re(a') at 10.0 GHz on substrate thickness is shown at several different temperatures for 25 μm wide strips. The relative dielectric constant, ε_r, is 10.5. The superconducting film parameters are the same as Fig. 3.

Fig. 5 at several different temperatures. The superconducting film parameters are the same as mentioned previously. As the substrate thickness decreases the percentage of energy stored in the superconducting films increases and hence the slowing factor increases.

CONCLUSIONS

The previous cases demonstrate some of the interesting and unusual properties that can be achieved by using very thin superconducting films in transmission line structures. With the discovery of new superconducting materials with transition temperatures above liquid nitrogen temperature there is likely to be much future interest in exploiting these materials for microwave applications, particularly transmission lines. The storage of energy in superconducting thin film transmission lines can be a major consideration when designing compact microwave circuits.

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